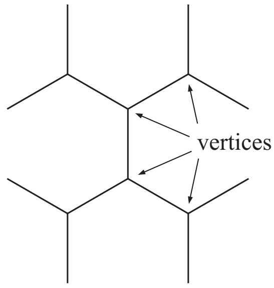
INVESTIGATION SOCCER BALLS

Soccer balls are constructed by stitching together regular pentagons and regular hexagons. They are therefore **semi-regular polyhedrons**.

Now if you look carefully at one of these balls, you will find it has *exactly* 12 pentagons. In this investigation we find out why.

Suppose our soccer ball is a polyhedron constructed from p pentagons and h hexagons. The faces always join at a vertex with two other faces.

For example,



1. Write down an expression for f, the number of faces in the graph of polyhedron, in terms of p and h.

2. **a** How many edges do the pentagons have in total?
   b How many edges do the hexagons have in total?
   c How many edges does the graph of the polyhedron have in total? Call this number e. Be careful to count each edge only once.

3. Given that each face meets with two other faces at a vertex, find a formula for V, the total number of vertices of the graph of the polyhedron.

4. Use Euler's rule to complete the proof.

Note that there is no restriction on the number of hexagons. In fact, we do not need to use any. What shape would we obtain if we used only pentagons?

Extension questions:

1. If we used pentagons and squares, would we end up with 12 pentagons and an unrestricted number of squares?

2. Prove that a sphere cannot be "tiled" by hexagons alone with the hexagons overlapping.